## We studied Linear Dynamical Systems

The dynamical system is **fecursive**, output = new input, The systems <u>usually find</u> an orbit where the input is equal to the output, this

#### is an equilibrium point.

Behavior is always changing unless it has reached its equilibrium point

### **INPUT = OUTPUT**

Recursive systems represent **nature**,

#### Example: an animal population is dependent on parameters

of growth and the previous years population.

# We studied the nature of Chaotic Dynamical Systems

Same as linear systems, with larger parameters.

These are very sensitive systems, a small change may change the entire behavior.

## Raising parameters: periodic, eventually chaotic

Chaotic systems jump around and <u>don't</u> settle down to a cycle like a Linear Dynamical System.

# We began by understanding simple harmonic motion.

Harmonic motion was an **integral part** of the more complex systems, so we set out to understand them.

Mass hanging from a string

Equilibrium is where the spring's force is equal to gravity.

The mass is displaced downward, so that the force of the spring exceeds the force of gravity, therefore acceleration upward. Mass passes equilibrium, and the process is repeated.

Graph describes the motion of harmonic motion.

# We originally attempted to describe a spring cylinder system.

Vertical energy is transformed into rotational (torsional) energy. When torsional is max, vertical is min.

Proved too difficult to measure using available instruments.

# We found a system whose motion was analogous to the cylinder system.

Two masses hanging from a pole, connected by a spring. Coupled oscillator: Movement of one depends on the other. Shows the energy transfer inherent in the cylinder system.

### System Setup

Two masses tracked by distance probes, measure distance to nearest object.

K= a constant that helps describe the string's reluctance to stretching and contracting.

Displaced both in one direction

Displaced both in opposite direction

Displaced one towards the probe.

### Both displaced in one direction: Parallel movement.

We can see that when mass is far away from the probe, the other is close, and then their positions reverse.

### The "Omega" is a constant that describes the

period of the oscillation. It is specific to a system.

## One displaced (Mass 1)

We see here that after mass one is released and begins to seek equilibrium, the spring transfers some of its energy to Mass two.

The height of this point is where Mass 1 is relatively inert, and mass two has a strong oscillation.

The opposite occurs, and this repeats.

# Now we relate the two, and describe one in terms of the other.

We see that one of the masses in the double pendulum represents vertical movement in the cylinder system, and the other represents rotational movement.

We see that although the oscillators have different periods, one can still represent the other because they are both similar coupled oscillators. As the springs extend and contract, they transfer a slight amount of torsional energy, this is what is seen in the system.