UNCW 2010

SUMMER VENTURES IN SCIENCE & MATHEMATICS

Nonlinearity and Chaos in Physical Phenomena

Computer applications in Physics

Christian P. Loftin

The Early College at Guilford

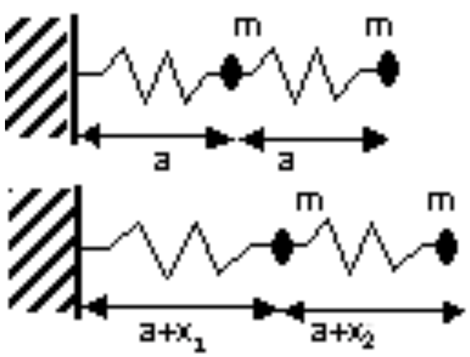
Joe Puccio

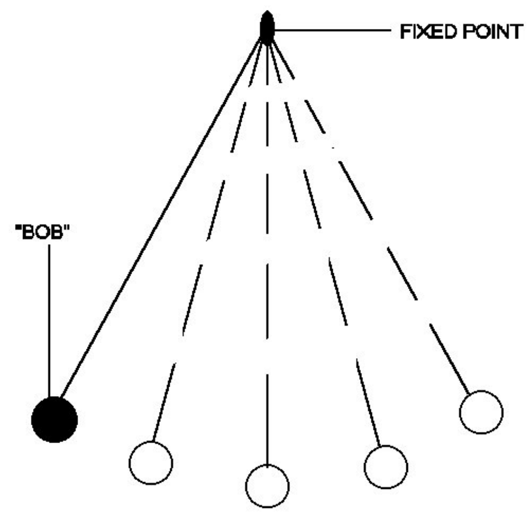
2010

Table of contents

Linear Dynamical Systems

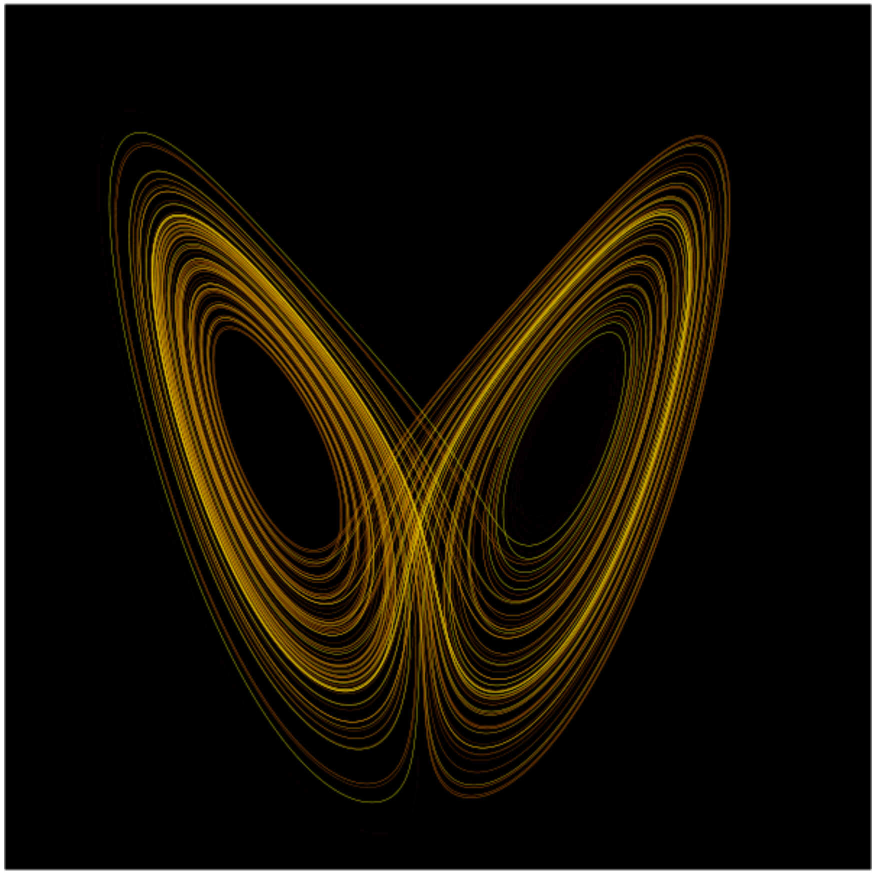
- Constantly evolving behavior
- Small changes in initial conditions → similar orbits
- Typically converges to a cycle
- Examples: pendulum, spring





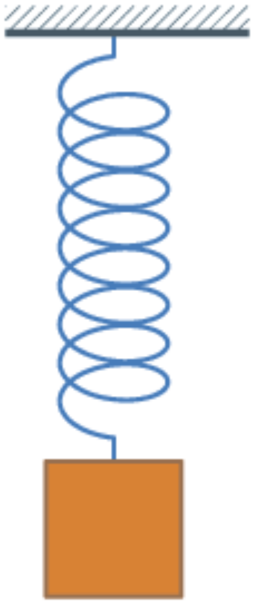
Chaotic Dynamical System

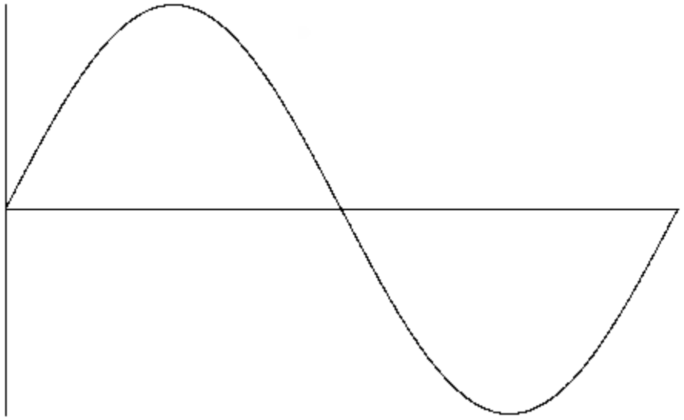
- Sensitive dependence on initial conditions
- Change parameter leads to bifurcations and chaos
- Often exhibits strange attractors
- examples
  - Weather
  - Stock market



Harmonic Oscillators

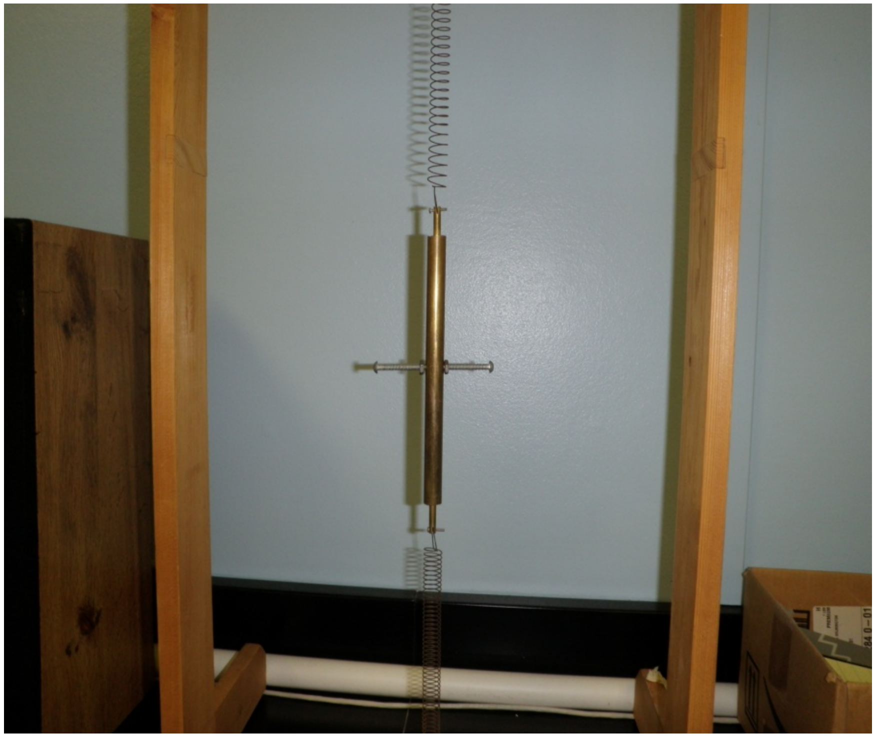
- Oscillator: repetitive motion about a central value
- Similar motion seen in Pendula and Spring systems
- Sinusoidal behavior





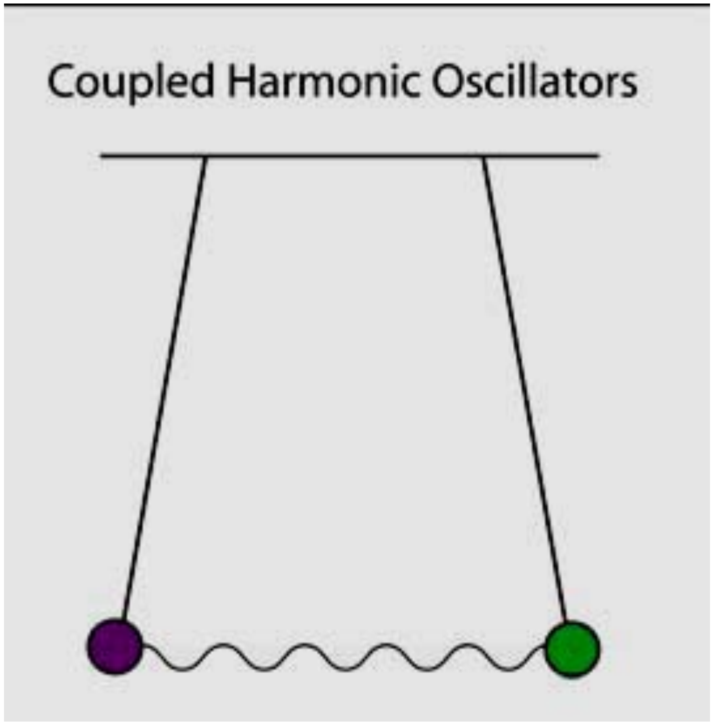
The Spring-Cylinder system

- Cylinder connected to a frame by springs on top and bottom
- Springs move vertically while cylinder rotates
- “beats”
- Difficult to measure
  - Use simpler model



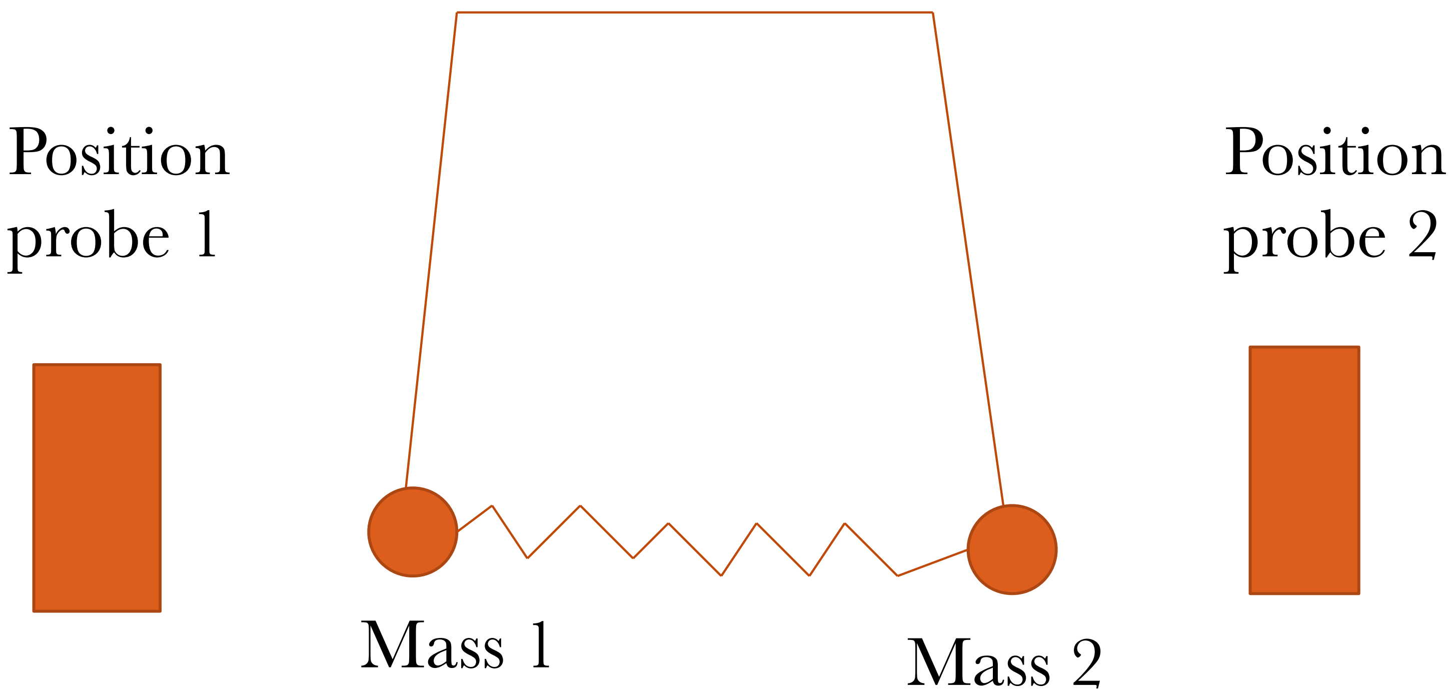
Double Pendulum

- Exemplifies transfer of energy within an oscillator
- Coupled oscillator
- Double pendulum motion analogous to Cylinder System



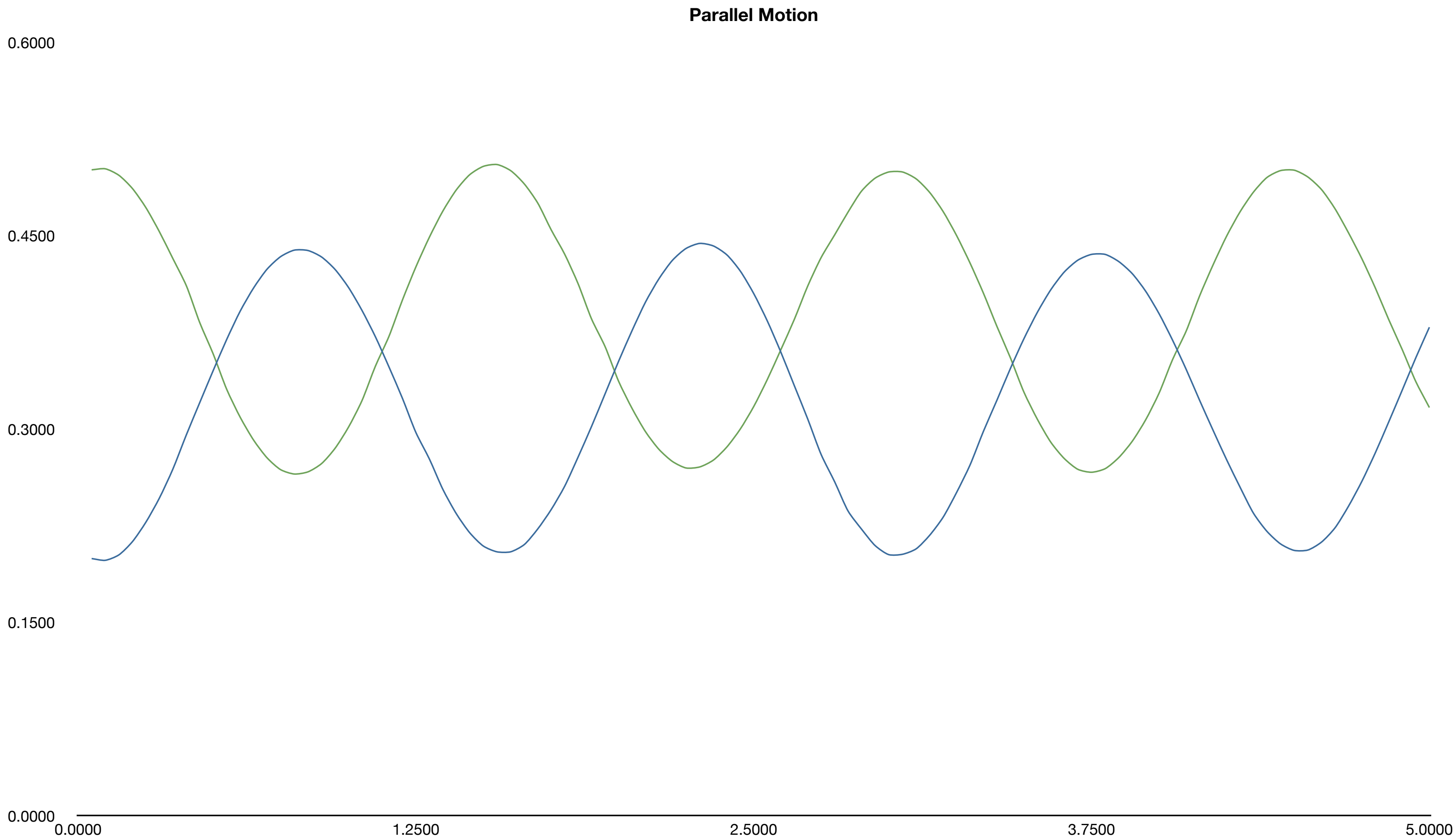
Setup

- Variable number of springs connecting pendula
- Ultrasonic distance probes record position

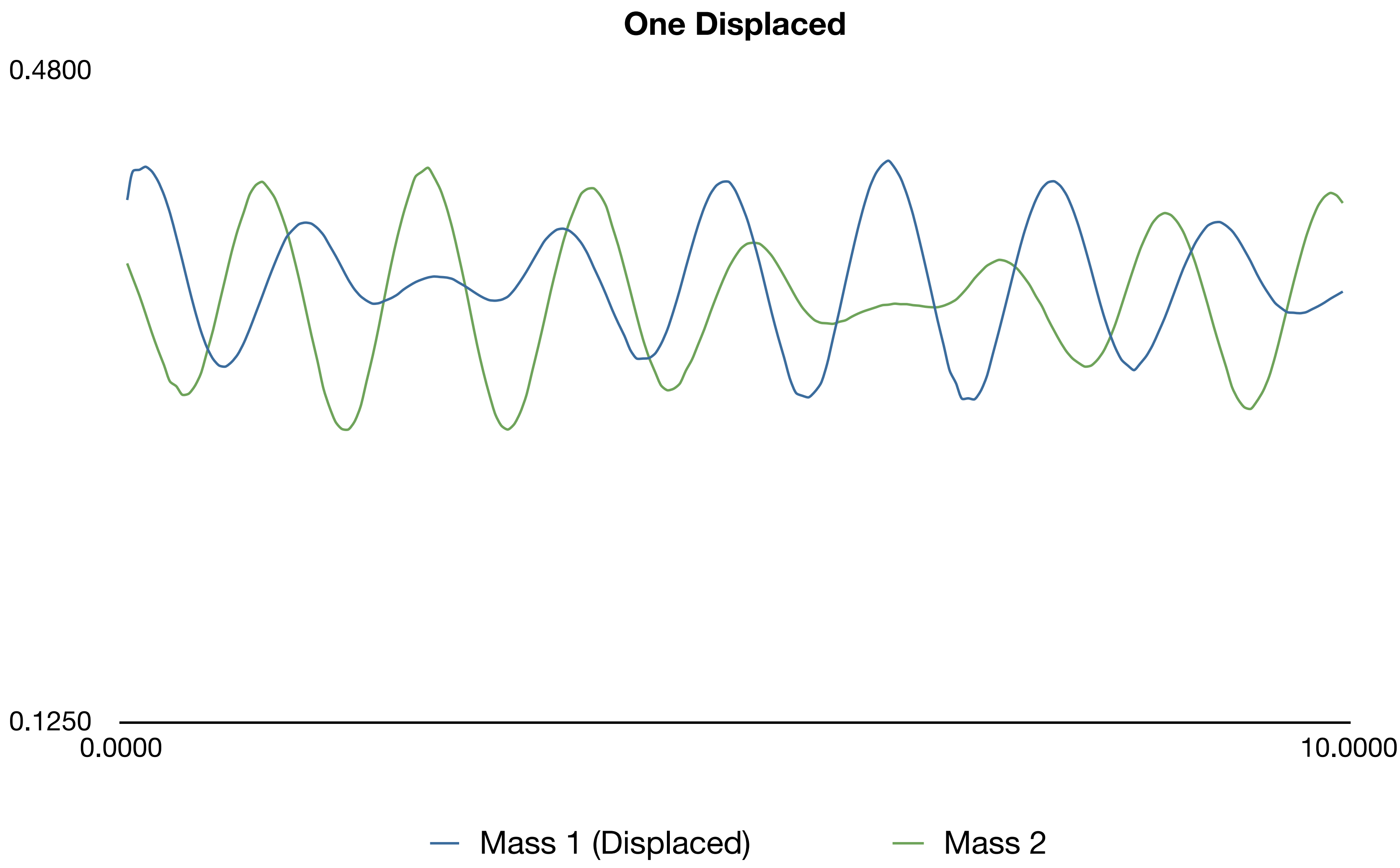


Motion in-phase

Mass 1 Mass 2

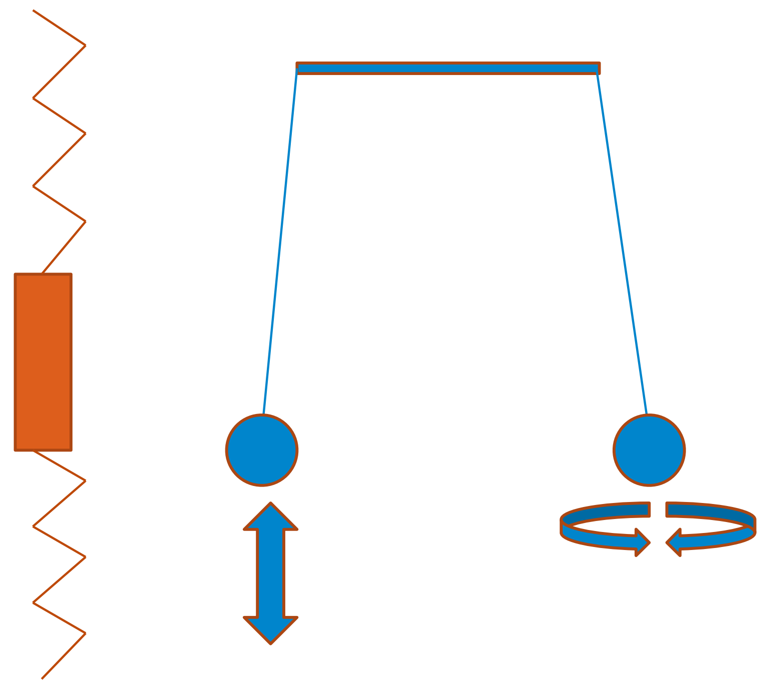


Phase shift

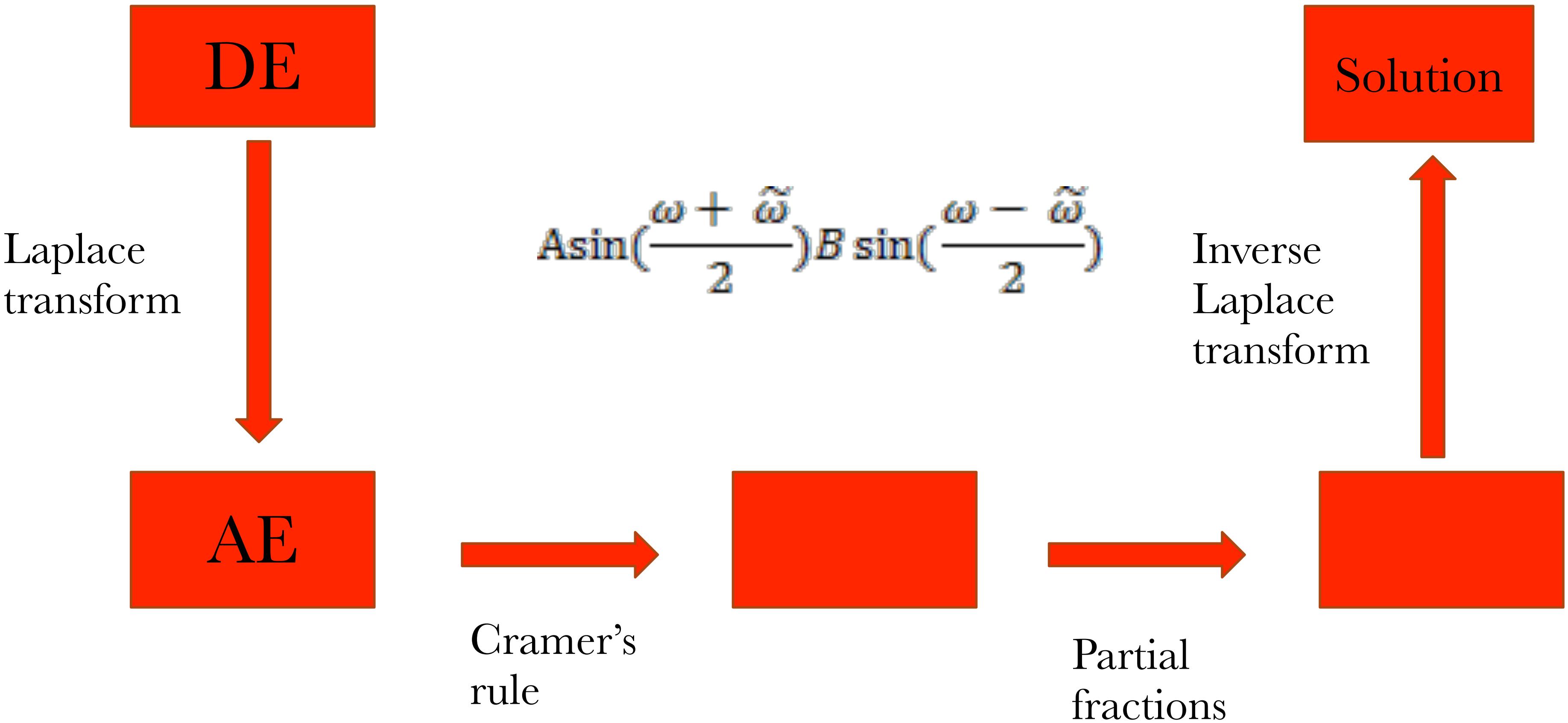

One Displaced
0.4800
0.1250
0.0000
10.0000
Mass 1 (Displaced)
Mass 2

Cylinder System

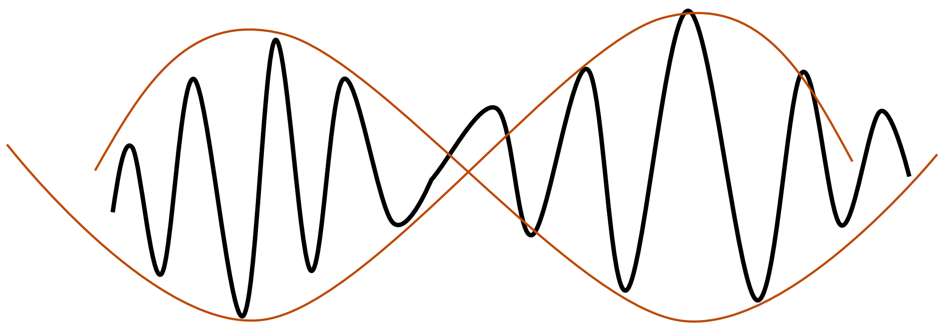
- One mass displaced in Pendulum represents Cylinder System
- When one mass' movement maximum, other minimum
- When rotation of Cylinder maximum, vertical minimum
- Transformation of movement from one mass to other same as rotational energy to vertical



Equations of motion for masses

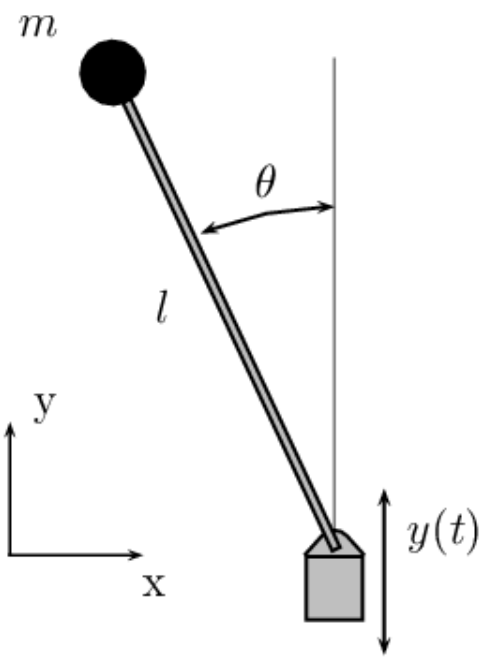


Beats



The Driven Inverted Pendulum

- Unstable equilibrium
- Driven by springs connected to a SHO
  - Nonlinear
  - Duffing dynamical system



Behavior of Duffing Systems

- We begin with unforced system

$$ax'' + bx' + cx + dx^3$$

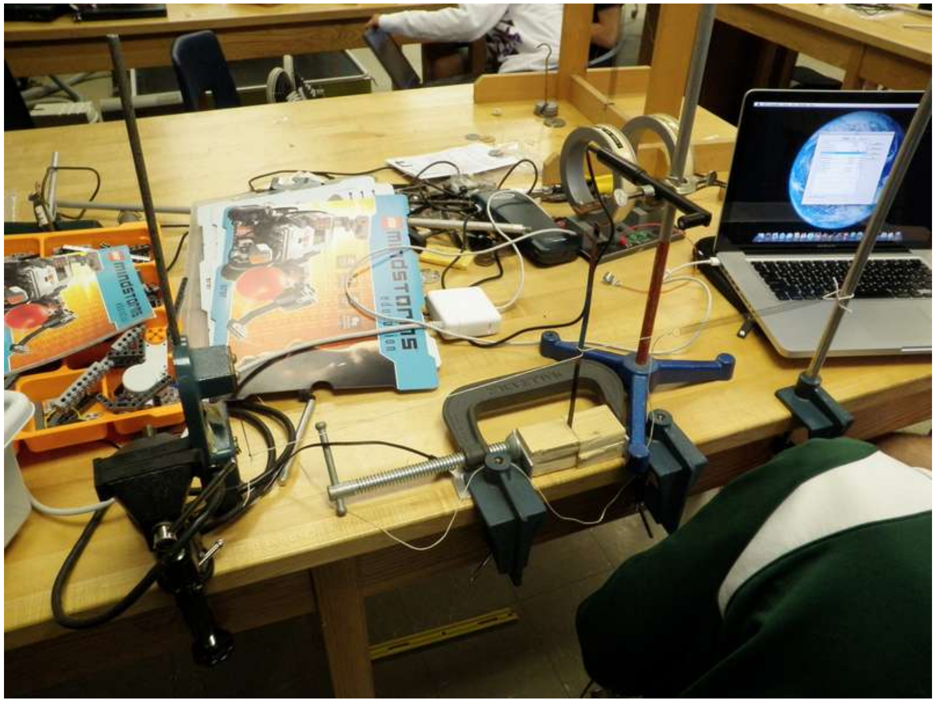
- Leads to Eigenvalue problem

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$$

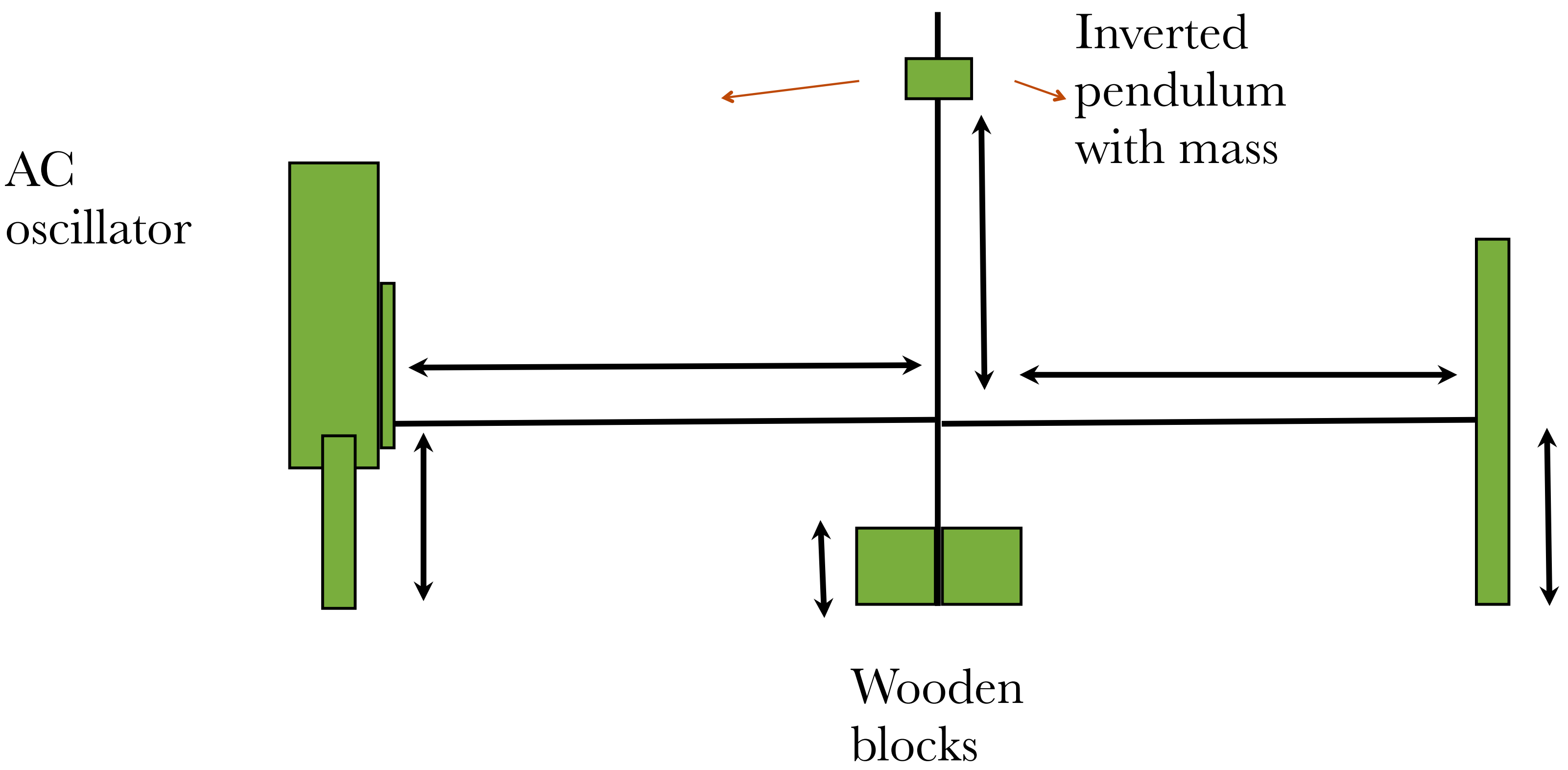
- Quadratic equation for λ
- Case λ = complex

The AC oscillator

- Initial test using AC (alternating current)
  - 60 hertz
  - String connected to one of two nodes



setup



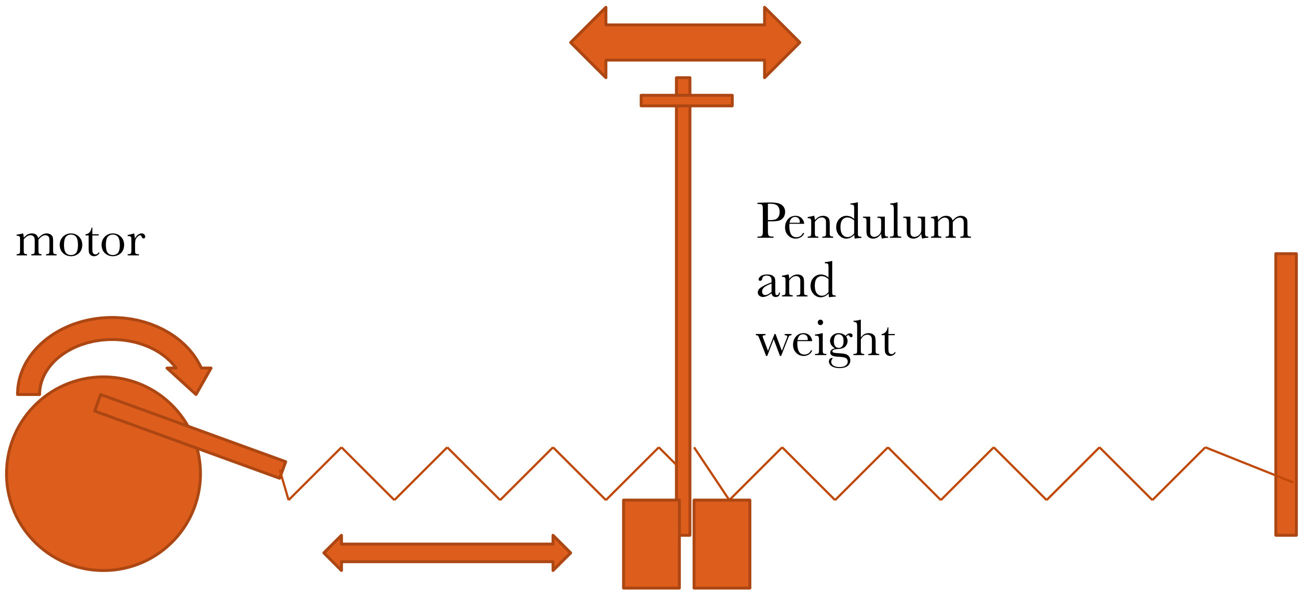
Magnetic Field vs. time (no magnets)

With two magnets

The Motor Driven Oscillator

- Motor provides a slower driving force with larger amplitude
  - adjustable
- Nonlinear term becomes large
- Weight at top of pendulum forces the pendulum into a two-well mode
  - Stable equilibrium points on left and right
  - Unstable at center

Setup



Data

Damped motion

Free motion

Data (cont.)

"beats"

Analysis

- Driving force produces far more irregular behavior
- Motor acts as Nonlinear term
- With no motor, acts as a simple oscillator
- Left and right equilibrium points are stable, center is unstable (repelling)

Citations/ acknowledgements

A first course in Chaotic Dynamical systems-Robert Devaney

An introduction to Chaotic Dynamical systems-Robert Devaney

Nonlinear Physics with Maple for scientists and engineers-Richard Enns, George McGuire

Special thanks to:

Dr. Lugo

Mr. Cavender

Dr. Herman

UNCW department of physics

SVSM staff