Introduction

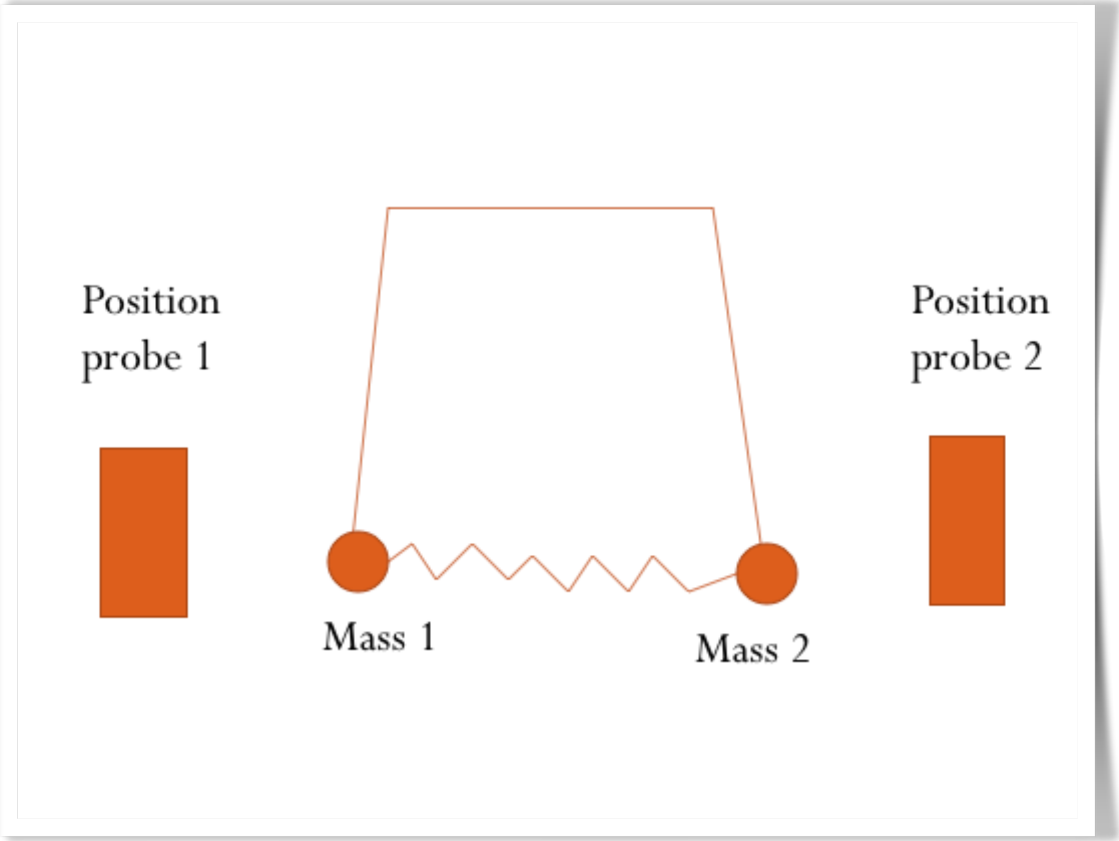
Dynamical Systems

In the research prior to our experimentation, we studied many systems, one of which was a Linear Dynamical System. These systems are deterministic, which means that for one input or system-state, only one output can arise. These systems are in constant evolution because the input is constantly changing due to the fact that they are recursive systems (output gets fed in as new input). This is remarkably important because they represents natural systems. If one were to change the initial conditions slightly, the new behavior would be very similar to the previous behavior because the function guiding the mechanics are so simple. Quite often, the system will find an orbit, that is an input that when iterated will eventually lead back to that same input (we call the amount of time or number of iterations for this to occur the “period”).

Chaotic Dynamical Systems

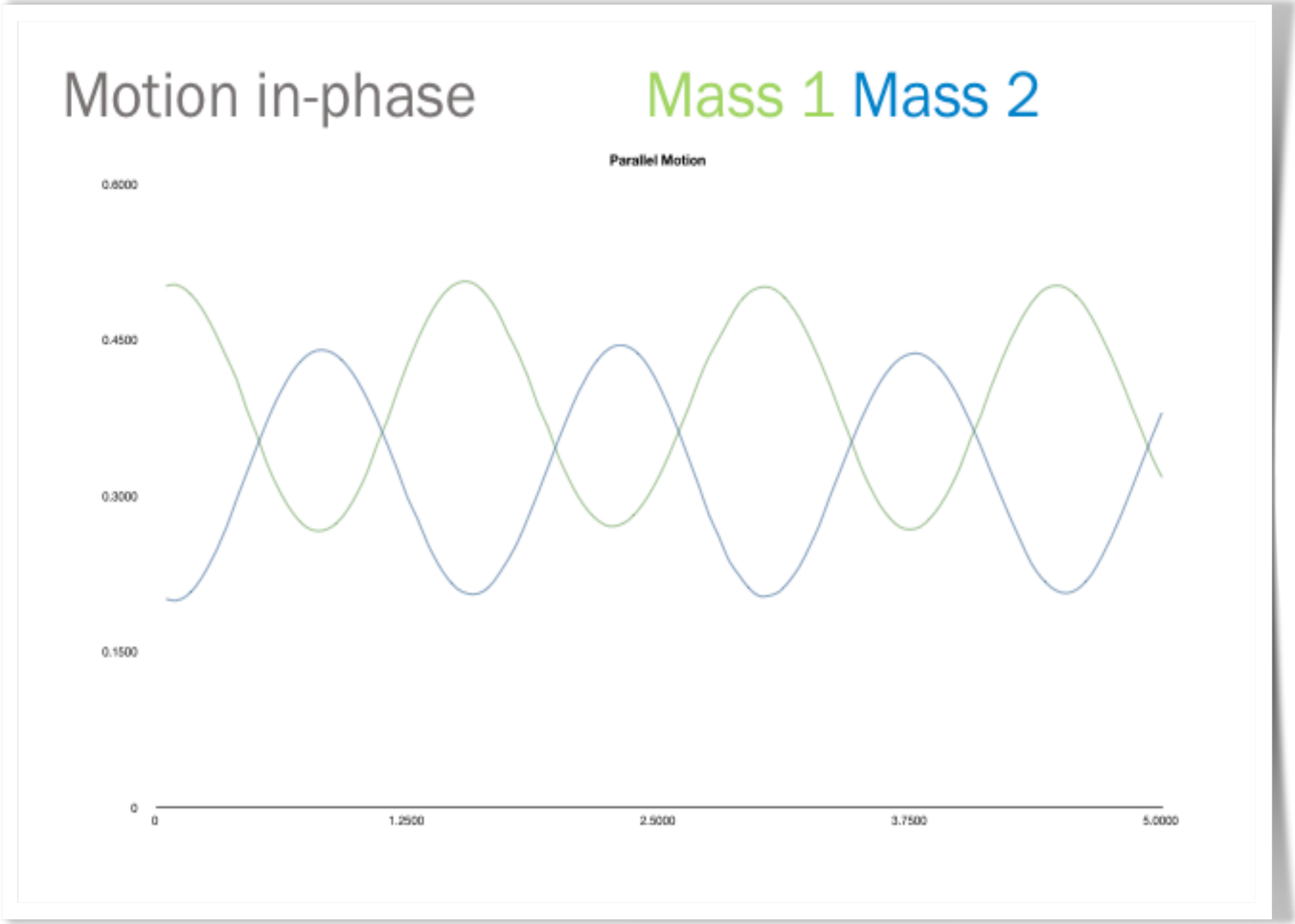
Unlike Linear Dynamical Systems, Chaotic Dynamical systems are very sensitive. They do not always exhibit the same narrowing in on an orbit like Linear systems. They love to jump around. Because they’re so volatile, a small change in what was originally inputed may change the entire behavior of the system. We call this sensitive dependence on initial conditions. This is observed because a small change can build on itself as the function is iterated. At lower parameters such as a constant and small driving force, or a system where the feedback from the system is relatively negligible, the system may converge on one point because the driving force overrides the small feedback and all that is seen is the system’s reaction to simple driving factor. As the parameters are raised however and the system’s behavior becomes more dependent on its own feedback to the driving force combined with the driving force, we begin to see Chaos ensue. For instance, if in a pool of water one of the walls of the pool is oscillating at a constant rate and causing ripples in the water that travel to the opposite side of the pool. These ripples bounce off the opposite wall and combine with the incoming ripples to create secondary feedback. This, in turn, affects the other incoming waves, once again changing the behavior or the ripples until the movement is essentially Chaotic. Our objective was to understand these dynamics in the simplest of cases.

Double Pendulum



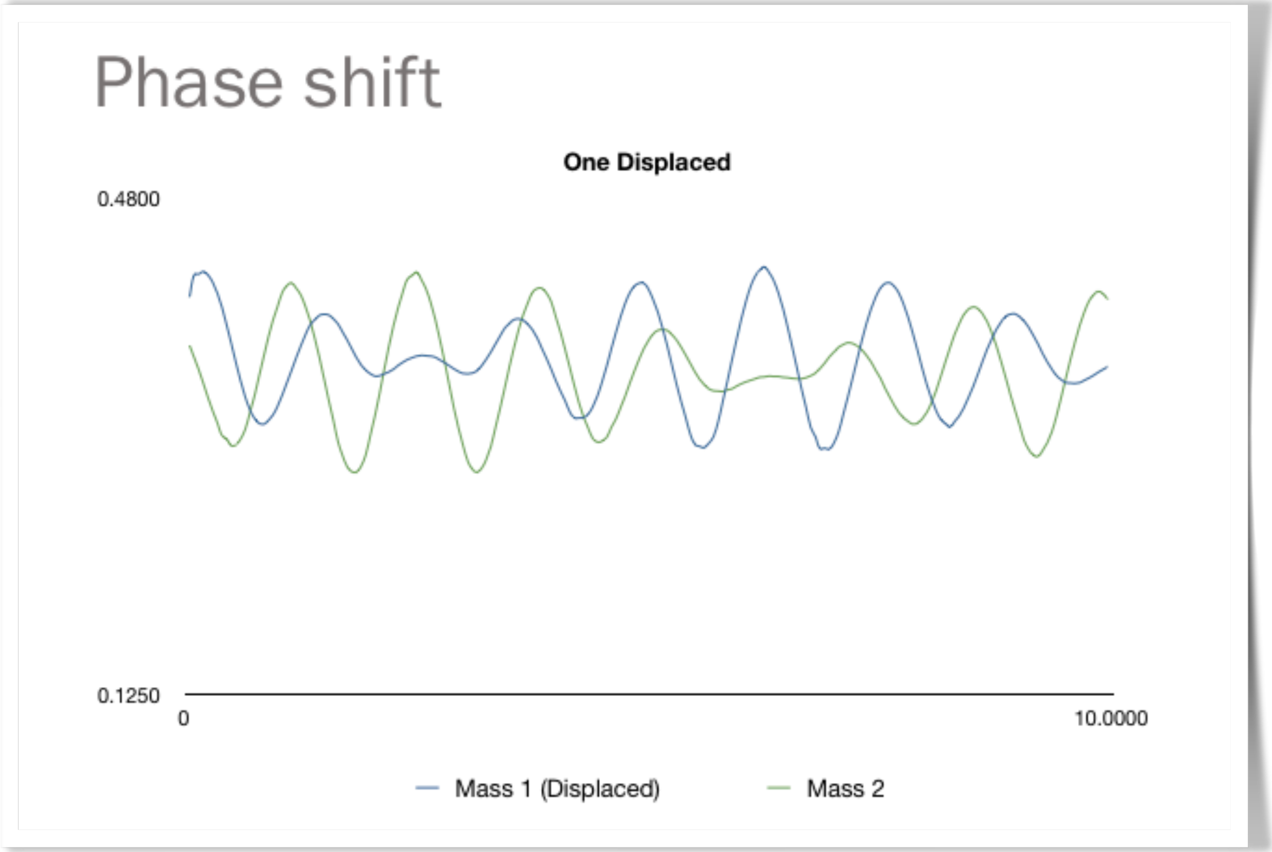
Design

We had two masses suspended from a bar with a spring connecting them. On the same plane as these masses, we had two probes that used sound waves to measure the distance between them and the closest object that obstructed these waves. We conducted many experiments with this setup, one in which we displaced both masses in opposite directions, one in which we displaced both masses in the same direction, and one in which we displaced one ball away from the other. The data from the latter two of these experiments is detailed below.



Parallel Motion

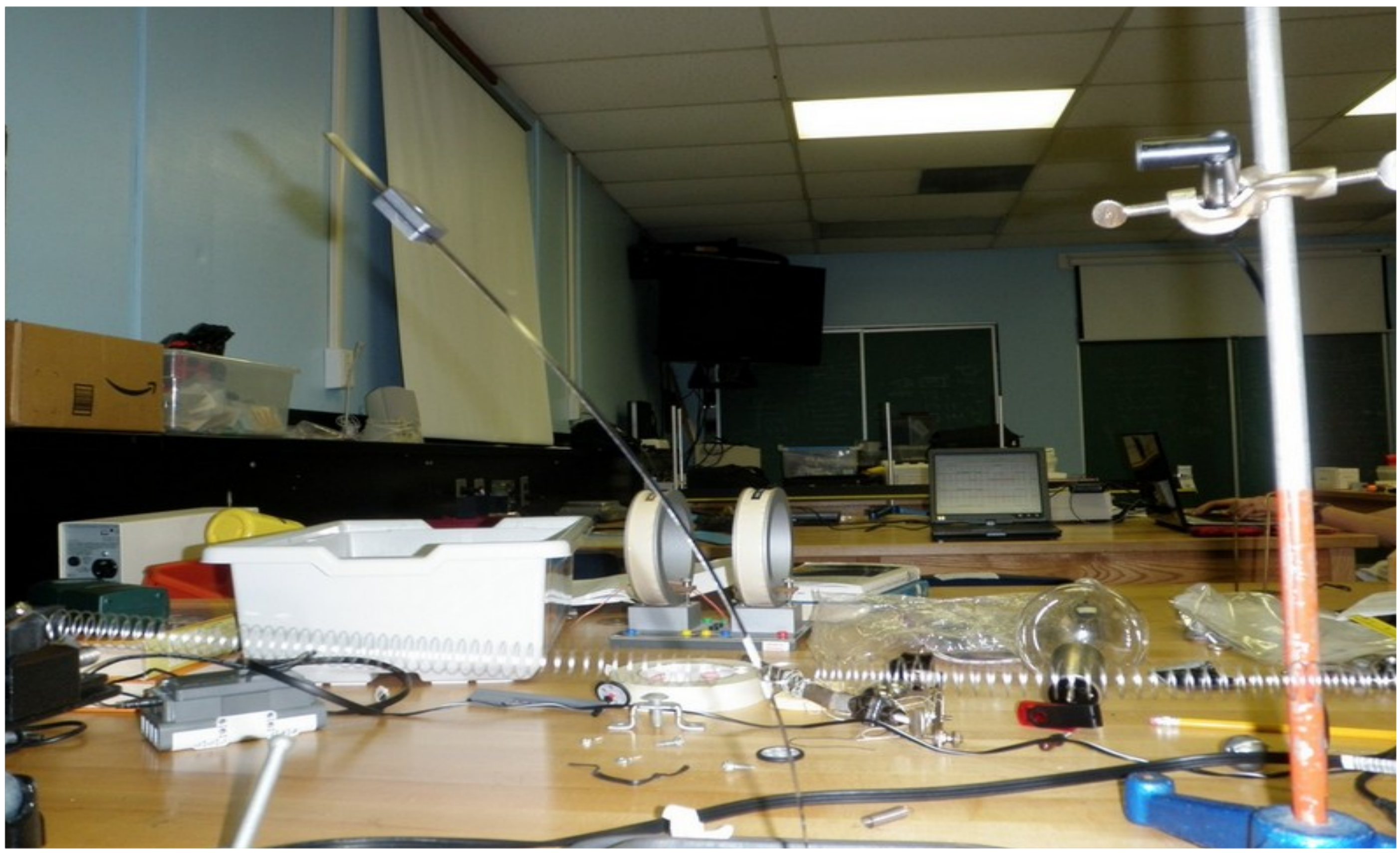
In this very simple case, both masses are displaced from their equilibrium by the same amount in the same direction. Mass one began farther away from its probe, and mass two an equal distance closer to its probe. When the masses were released, they fell toward their equilibrium points. Each mass' motion was independent of the other because they were not displaced relative to each other and no forces in the system caused them to become displaced relative to each other and therefore the spring was neither stretched nor compressed. Each mass oscillates about its equilibrium position like a simple harmonic oscillator.



Phase Shift

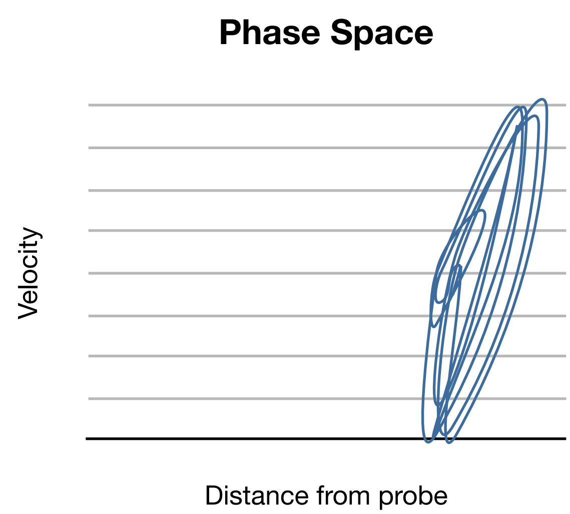
In this experiment, only one of the masses (mass 1) was displaced from its equilibrium. When released, it began to fall back toward its equilibrium. However, because the mass had acquired velocity, it overshot its equilibrium and compressed the spring slightly and thus displaced the second mass. The first mass then continued to oscillate about its equilibrium and transfer its remaining energy to the second mass. Once all of the energy had been transferred and the first mass was inert, the second was at maximum kinetic energy and thus in full swing. This cycle continues. It's an example of a coupled oscillator, which means the velocity and location of one of the masses depends on the other. If one were to isolate the motion of one of the masses, we can see alternating periods of maximal and minimal movement. The envelope of this motion is sinusoidal in form and referred to as a "beat."

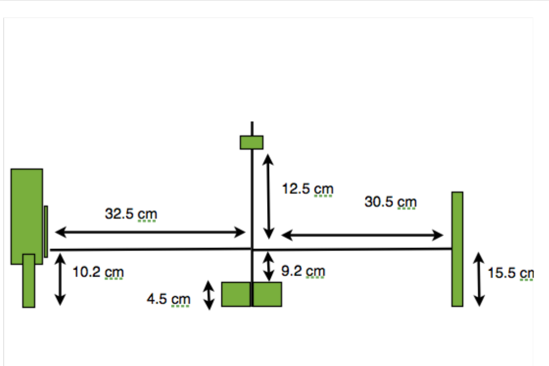
Driven Inverted Pendulum

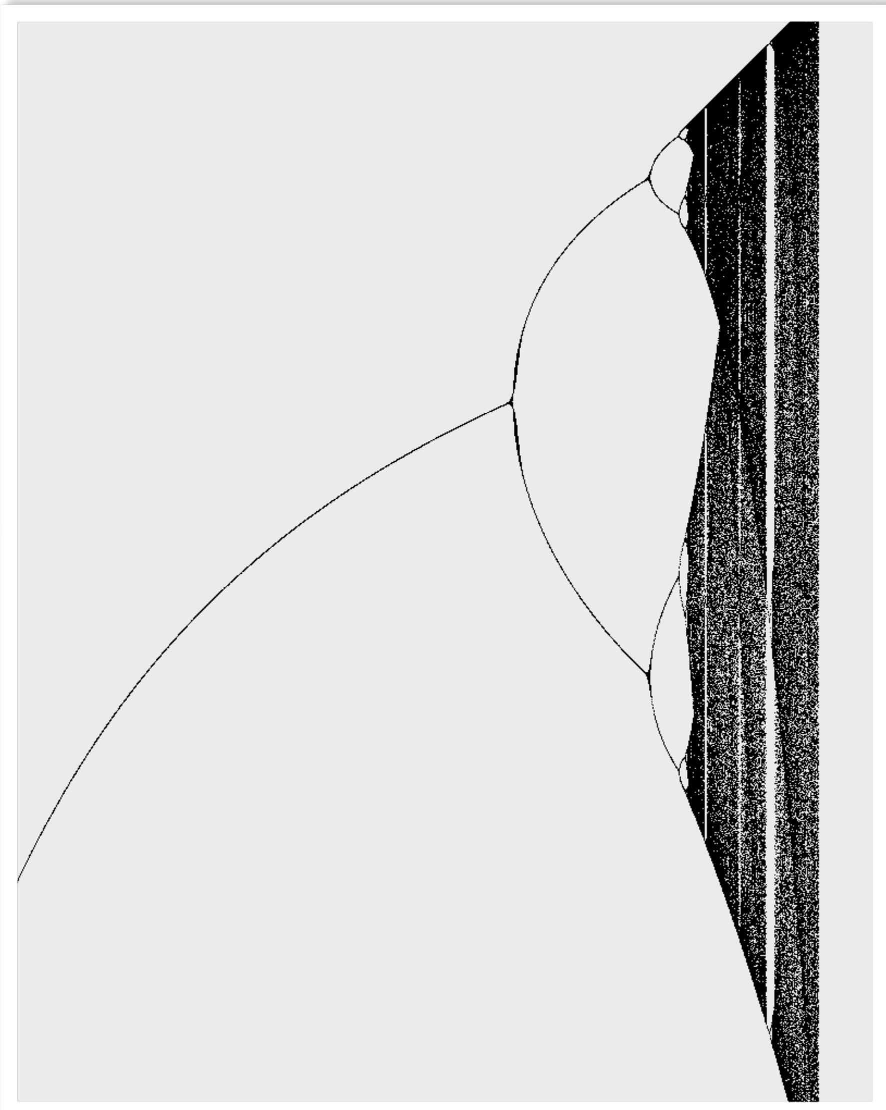


Design

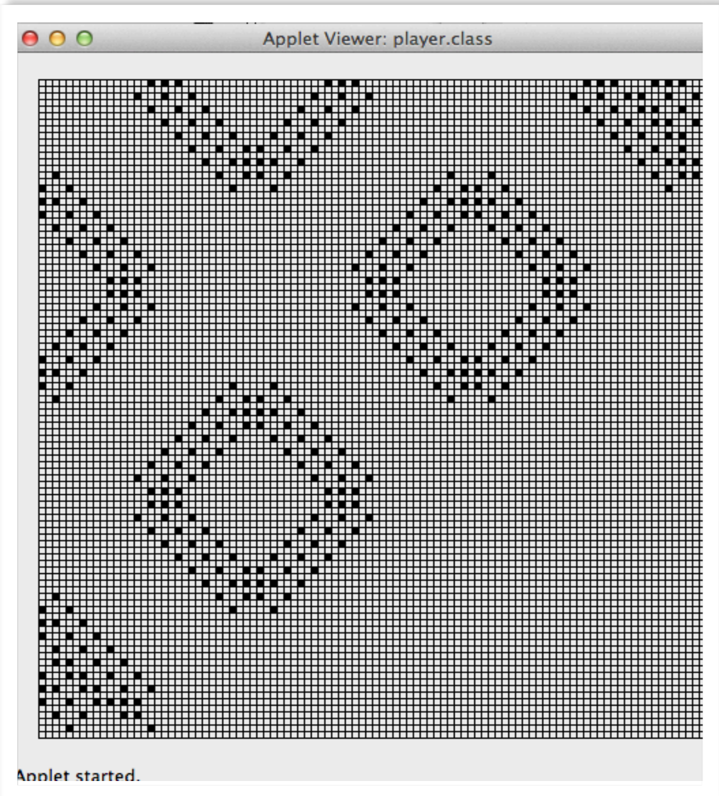
Attached to either side of a metal strip clamped between two blocks of wood were springs, one of which was connected to a motor whose rotation rate and amplitude we could finely control. Near the trajectory of the pendulum we placed a magnetic probe (pictured in the upper right) that would tell us the magnetic field strength at that point at any given time; because we had a magnet attached to the top of the pendulum, we would use this data to infer the position of the probe. Among the most interesting pieces of data we collected were Phase Space plots. These plot velocity against distance on a coordinate plane. We used this plot because it delivers an enormous amount of information about the nature of the system as we can think of any

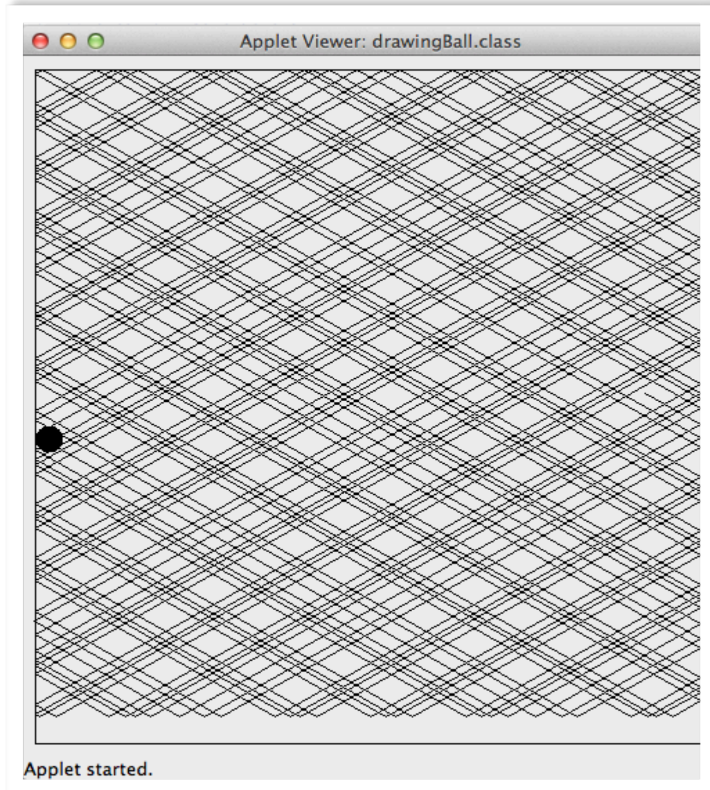


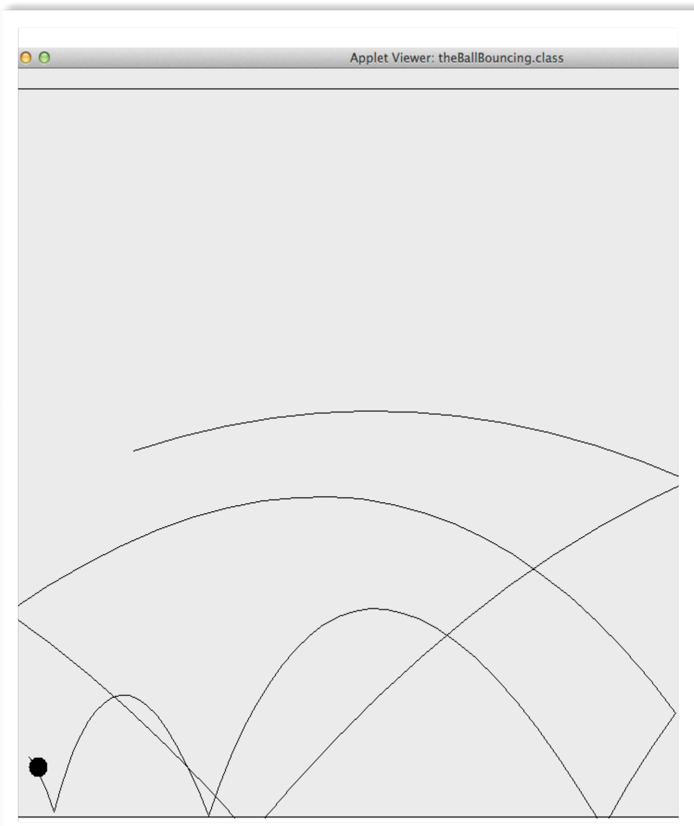




ordered pair as a system-state.







Bifurcation Diagram

One of the most fascinating aspects of this project to me were the formulaic analogs to the systems we were studying. Pictured here is the output of a Java program I wrote to generate what's called a bifurcation diagram. This is generated by the incredibly simple equation output = k(input-input^2). Moving from left to right in Th.e image, we vary the k parameter whose domain are real numbers between 1 and 5. For each of these k values, we choose any starting input between 0 and 1 (the diagram is resistive to the starting input) and feed the function's output for each calculation in as its next input. For many small k values, we see that the output settles to a singular state in the range of 0 to 1 where the function's input is equal to its output. The point to which this recursively defined function settles steadily increases until an amazing thing happens: it bifurcates. At this point, k is approximately equal to 3.03. The system beings to alternate between two states with a regular period for another large stretch of k values. Then, another bifurcation, and another, and another, until the state of the system jumps into restlessness where it does not settle down to any point or set of points. We call this Chaos. This is seen on the diagram as a smear of black. Within this Chaotic period lies transient moments of tranquility where the function finally finds a few points to settle on, however, this is short-lived as it inevitably erupts once again into Chaos. Remarkably, period doubling such as this is observed in physical dynamical systems.

Further Investigations

I've dealt a lot with dynamical systems since Summer Ventures. I've made Java programs that create Cellular Automata (upper left), model the motion of a ball bouncing within a box without the effects of gravity or friction (above), and model the motion of a ball subject to such forces (left).