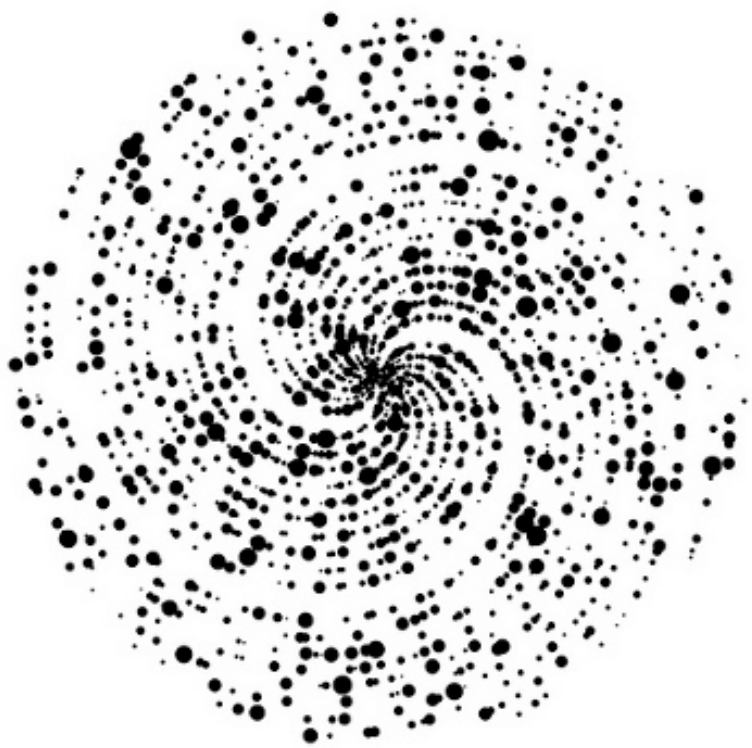
PRIME FASCINATION



An exploration of prime numbers

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Introduction

Prime numbers have fascinated mathematicians and puzzle solvers for millennia. This fascination is driven primarily by the elusiveness of primes to investigation. Currently, there exists no equation, method, or other form of determination that can produce the set of all primes. The only known way to link every prime is by its very definition: any positive integer that can only be evenly divided by one and itself. Primes are ubiquitous (in fact, it has been proven that there are infinitely many), and it's a combination of their simplicity of description, elusive capacity, and strange occurrences in nature that has sustained their study's zest over thousands of years.

With scrupulous observation of primes, we are able to discover possible routes to understanding, and perhaps contribute to a monumental effort to explain this mystery of the universe. It is with the conglomeration of these minute observations and explanations that we may come to a comprehensive understanding of nature's most remarkable creation.

What is a prime?

Primes are numbers that have a certain property called *primality*. When deciphering what numbers have this property, we refer only to positive integers. The set (collection of objects) of integers (from the Latin *integer*, which translates to "untouched" and hence undivided, whole)[1] is formed by the union (merging of multiple sets, removing duplicates) of the natural numbers (also called the "countable numbers" as they are used when counting) with 0 included, and their negative counterparts. These together form the integers (often denoted with a boldface **Z** which stands for *Zahlen* which is German for *numbers*)[2]. For the investigation of primes, we limit our consideration to those integers that are positive. This leaves us with $\mathbf{Z}_+ = \{1,2,3,4,5\ldots\}$. Within this set, $\mathbf{Z}_+$, lie our objects of interest: primes (often denoted **P**). We denote the containment of prime numbers in the set of positive integers:

$$\mathbf{P} \subseteq \mathbf{Z}_+$$

This tells us that for every element or object in the set **P**, this object exists in $\mathbf{Z}_+$. Now, how is it that we can come to this set **P** from the set $\mathbf{Z}_+$? Well, we must first define what it means for a number to be prime: a prime number is a number in $\mathbf{Z}_+$ that has no positive divisors other than 1 and itself. From this definition, we may begin to populate the set of **P** with elements. There are many ways of doing this. We call the mechanisms that attempt these tasks *algorithms.* Some of these algorithms test the divisibility of each number in $\mathbf{Z}_+$ to see if it is divisible by any number other than one and itself that is also in $\mathbf{Z}_+$ (e.g. for 4, is it divisible by 2 or 3?), and if it is not, the number is placed in **P**. This is often referred to as a "brute-force" approach as it tests every possible case without tact. We will discuss those algorithms that apply the definition of primes to chip away non-primes from $\mathbf{Z}_+$ so that only prime numbers remain.

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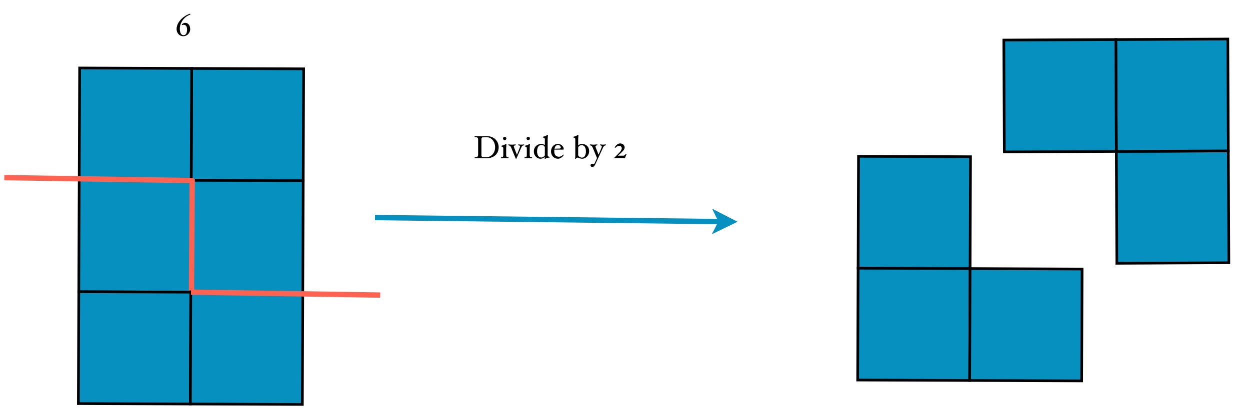
[1] "integer." Dictionary.com Unabridged. Random House, Inc. 22 Mar. 2012. <Dictionary.com http://dictionary.reference.com/browse/integer>.

[2] "Zahlen." The American Heritage® Dictionary of the English Language, Fourth Edition. 2003. Houghton Mifflin Company 22 Mar. 2012http://www.thefreedictionary.com/Zahlen

Divisibility

Before exploring the mechanisms by which we find the set of primes, we must understand the methods by which these mechanisms operate. In the case of determining prime numbers, the primary method is the testing the property of divisibility. If a number, a, can be split into b equal parts, then it is said that b divides a (this is denoted $b|a$). For example, if we choose a to be 6, and b to be 2, we find that $b|a$ because 6 can be split into 2 equal parts (namely, two parts each being three). If we instead chose a to be 5, we would find that $b \nmid a$ as we cannot split 5 in two equal parts. The property of divisibility can be explored visually; simply, but powerfully:



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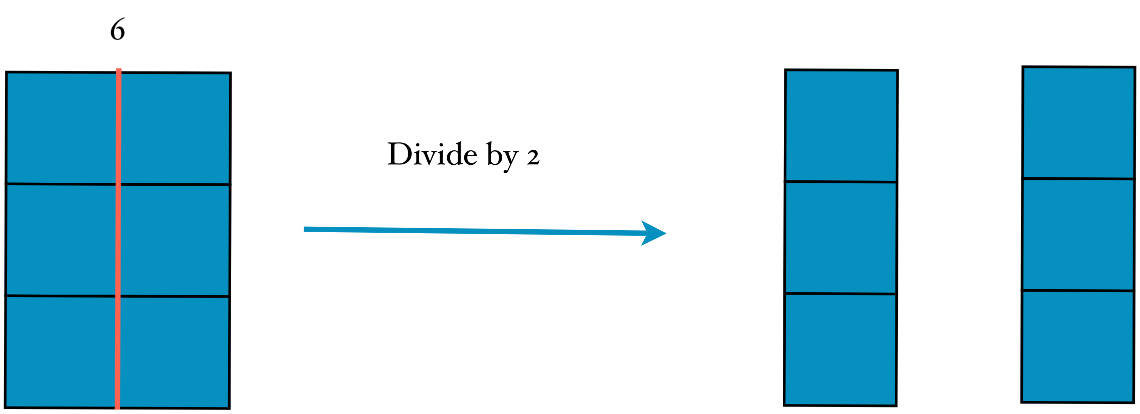

This visual representation of divisibility may enable future insights in understanding the distribution of prime numbers within the set of integers.

Populating $\mathbf{P}$

One algorithm used to find primes which is very similar to the ancient *sieve of Eratosthenes* algorithm accredited to Eratosthenes of Cyrene, an ancient Greek mathematician,[3] uses the modulo operator to filter through $\mathbf{Z}_+$ and find primes. The modulo operator is similar to the divisor operator in that it has two arguments (takes two inputs) and determines whether a certain relationship between these two arguments exist, but differs in its output (the modulo operator has a numerical output while the divisor operator has a boolean output (true or false)). In fact, the modulo operator can serve the function of the divisor operator and, in certain circumstances, provide more information about the relationship between the inputs.

The modulo operator returns the remainder when one number is divided by another. For example, 5 modulus 2 is equal to 1 (we divide 5 by 2 and we receive a remainder of 1). We can denote this operator as follows:

$$5 \% 2 = 1$$

(Note that this notation is more commonly used in Computer Science than Mathematics, the more common mathematical notation would be $5 \equiv 1 \pmod{2}$.)

3 Nicomachus, Introduction to Arithmetic, I, 13.

Aidian Twins

Two primes are said to be *Aidian Twins* if each of the primes, when added to their sum, is equal to a prime. This has currently been tested for thousands of consecutive primes, and it's interesting to note that the resultant pair of primes often happen to be consecutive as well. One supposition that's common when such a relation is found is: are there infinitely many Aidian Twins? Another open question is: what does the distribution of Aidian Twins look like? Is there a non-trivial property that all Aidian Twins share?

Distribution

We also investigate the number of primes between a range within $\mathbf{Z}_{+}$. For instance, we observe that the number of primes between the integers 1 and 100 is substantially less that those between 101 and 1000.

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Nicomachus, Introduction to Arithmetic, I, 13.