Aidian Primes

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Introduction

The seemingly patternless primes have been investigated by mathematicians for hundreds of years. While little advancement has been made on uncovering a pattern that connects all primes, explorations of primes of certain forms have been more fruitful. Each prime-type is a subset of the prime numbers that match some pattern or follow a rule. Some examples are Mersenne primes which are all primes of the form $2^n - 1$, twin primes which are all primes p such that $p + 2$ is also prime (similarly, cousin primes are equivalent except $p + 4$ and sexy primes are equivalent except $p + 6$), and Wagstaff primes are all primes p such that $p = \frac{2^q+1}{3}$ where q is another prime. We introduce yet another subset of primes, Aidian primes: all primes r such that $r = 2p + q$ where p and q are consecutive primes.

Exploration

The discovering example of Aidian primes was found in the room number 385. Both 3 and 5 are prime and their sum is the center number, 8. Additionally, $3 + 8 = 11$ is prime as is $5 + 8 = 13$. Thus, this example demonstrates that 13 and 11 are Aidian primes as they can each be written in the form stated above with p and q consecutive primes: $13 = 2(5) + 3$ and $11 = 2(3) + 5$. This example, while not alone in this quality, is somewhat unique as this choice of p and q could result in the production of two Aidian primes and the two primes produced are also consecutive.

The following conjectures arise quite naturally from the definition and our example: What fraction of Aidian primes have a neighboring Aidian prime? Is this proportion constant? If not, does it have a limiting value? Does every Aidian prime have a unique p, q pair? If two primes are yielded from this operation, are they necessarily consecutive? Are there infinitely many primes of this form? What is the distribution of Aidian primes within the integers? What if the restriction that p and q be consecutive is dropped? What if we consider all primes that can be written as a linear combination of another two, that is: $r = np + mq$ where n, m are integers and p, q are consecutive primes? And the questions continue indefinitely. We will attempt to answer many of these questions and leave the rest to the investigation of the reader.

We will begin by answering the following: If two primes are yielded from this operation, are they necessarily consecutive? Fortunately, this only requires a counter example which can be provided easily with the use of a computer. One such counter example, which is believed to be the first, is $353 = 2(113)+127$ and $367 = 2(127)+113$. Between the primes 353 and 367 lies the prime 359. A follow-up question may be: can there be only at most 1 prime separating the resulting primes? A quick investigation also says no as 1481 and 1489 are Aidian primes resulting from $p, q = 491, 499$ and the primes 1483 and 1487 both lie between them.

We proceed by attempting to answer the next question which may also be easily satisfied by a counter example: does every Aidian prime have a unique p, q pair? Using a computer we found that out of the 5561 Aidian primes generated by p, q pairs taken from the first 25000 primes, all had a unique p, q pair. While this is not proof and does not by any means conclude investigation, it does suggest that each Aidian prime has a unique p, q pair. It may also be observed that the set of numbers resulting from inputs of consecutive primes p, q into $2p + q$ is necessarily going to be monotonically increasing, which allows us to conclude that each $r = 2p+q$ will be possible only from a distinct choice of p, q.

Next, we answer the slightly more difficult question: what fraction of Aidian primes have a neighboring Aidian prime? For each Aidian prime, we mark whether one of its neighboring primes is Aidian, or if neither are Aidian. After the first 1000 primes, the ratio is $\frac{16}{64} = 0.25$. After 5000, $\frac{35}{265} = 0.13$. 10000, $\frac{50}{471} = 0.106$. 30000, $\frac{113}{1245} = 0.09$. 60000, $\frac{231}{2281} = 0.10$. And after 75000 primes, the ratio is $\frac{277}{2765} = 0.1001$. Remarkably, it seems as though the ratio of Aidian primes with Aidian neighbors to isolated Aidian primes has a limiting value of $\frac{1}{10}$.

We then attempt to answer a larger question about distributions. How are the Aidian primes distributed amongst all the integers? Using a computer, we first generate a list of all Aidian primes up to a certain number (100000), then we cycle through all the Aidian primes, taking the inverse (in order to for a logarithmic fit) of the following ratio:

$$\frac{\text{Aidian Prime Size}}{\text{Count Of Aidian Primes So Far}}$$

for each Aidian prime. The ratio stated above gives us a sense of how many Aidian primes are less than a certain integer value. Plotting the data and using Excel to generate a treadline, we receive the following equation with an $R^2 = 0.975$:

$$y = 19.536 \ln(x) - 126.98$$

This suggests that the distribution of Aidian primes among the integers is "nice". That is, the change in their distribution between segments of the integers follows some pattern. More investigation should be conducted on this distribution.

Next, we examine the distribution of primes c such that $c = 2a + b$ where a, b be prime but not necessarily consecutive primes. Due to computational limitations, we were only able to vary $a, b \leq 7919$. When the list of resulting prime numbers is examined, it appears almost indistinguishable from the list of all prime numbers. In fact, the only primes missing from the list are 2, 3, and 5. Our results suggest that the distribution of primes of this form in the integers is equal that of all primes in the integers, and that a very interesting conjecture can be made: all primes greater than 5 can be written as $2p + q$ where p, q are prime.

Finally, we begin to explore whether there exist infinitely many Aidian primes. We may first get a sense for their cardinality by trying to determine if we can find a limit to the size of an Aidian prime given a large set of prime numbers. Because we have found Aidian primes as high as 1000000 and our prime number data would not permit any higher, we are inclined to believe that there is no obvious cap. We may also gain some intuition regarding their cardinality by getting a sense for whether the following series diverges:

$$\sum_{p=0}^{\infty} \frac{1}{A_p}$$

where A is the set of all Aidian Primes. We may do this by comparing our series to a series we know diverges, such as $\sum_{n=0}^{\infty} \frac{1}{100n}$. If we look at the ratio of corresponding terms, that is, $\frac{A_p}{100p}$ while p tends to infinity, and this ratio approaches 0, then this means that the values of $100p$ escape to infinity faster than those of A_p. Therefore, $\frac{1}{100p}$ goes to 0 faster than $\frac{1}{A_p}$, and because we know that $\sum_{p=0}^{\infty} \frac{1}{100p}$ diverges, then we should be able to conclude that $\sum_{p=0}^{\infty} \frac{1}{A_p}$ diverges. Using a computer to generate the first 6500 terms of $\frac{A_p}{100p}$, we unfortunately find that this ratio does not approach 0, and our test is inconclusive.