Homework 8

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Exercise 4.3.1

(a) Let $\epsilon > 0$, $|f(x) - f(c)| = |\sqrt[3]{x} - 0| < \epsilon$ so letting $\delta = \epsilon^3$ will ensure that $|x - 0| < \delta \Rightarrow |\sqrt[3]{x} - 0| < \epsilon$.

(b) Using the identity given in the problem, we can set $a = \sqrt[3]{x}$ and $b = \sqrt[3]{c}$ where c is the point being approached in the domain. Multiplying $|\sqrt[3]{x} - \sqrt[3]{c}|$ by $\frac{\sqrt[3]{x^2} + \sqrt[3]{x}\sqrt[3]{c} + \sqrt[3]{c^2}}{\sqrt[3]{x^2} + \sqrt[3]{x}\sqrt[3]{c} + \sqrt[3]{c^2}}$ yields $\frac{|x-c|}{\sqrt[3]{x^2} + \sqrt[3]{x}\sqrt[3]{c} + \sqrt[3]{c^2}} \leq \frac{|x-c|}{\sqrt[3]{c^2}}$ so setting $\delta = \sqrt[3]{c^2}\epsilon$ will ensure that if $|x - c| < \delta$ then $|\sqrt[3]{x} - \sqrt[3]{c}| < \epsilon$.

Exercise 4.3.3

Because $|ax + b - ac + b| = |ax - ac| = a|x - c|$ then $|x - c| < \frac{\epsilon}{a}$ for an arbitrary ϵ so setting $\delta = \frac{\epsilon}{a}$ will ensure that if $|x - c| < \delta$ then $|(ax + b) - (ac + b)| < \epsilon$.

Exercise 4.3.7

The set K defines the roots of the function $h(x)$.

Exercise 4.3.8

Exercise 4.3.9

(a) We know that $|f(x) - f(y)| \leq c|x - y|$ and we want to show that $|x - y| < \delta \Rightarrow |f(x) - f(y)| < \epsilon$. From $|f(x) - f(y)| \leq c|x - y|$ we can say $c|x - y| \leq \epsilon$ and $|x - y| < \frac{\epsilon}{c}$ so we set $\delta = \frac{\epsilon}{c}$ and we can be sure that $|x - y| < \delta \Rightarrow |f(x) - f(y)| < \epsilon$.

(b) Given an equation such that $|f(x) - f(y)| \leq c|x - y|$, we know that $f(x)$ has to be an equation of degree one with a slope $-1 < c < 1$. Because of this constraint, we know that a sequence defined recursively would be decreasing strictly and absolutely ($|y_{n+1}| < |y_n|$) Because of this condition, we know that