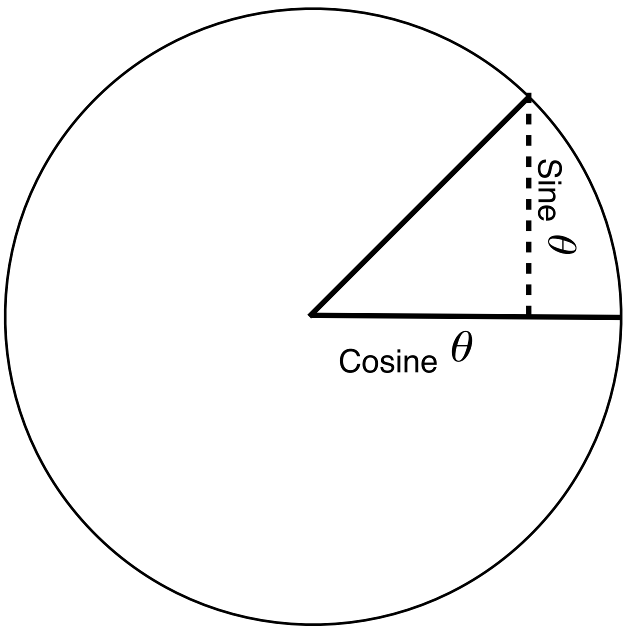
The Problem

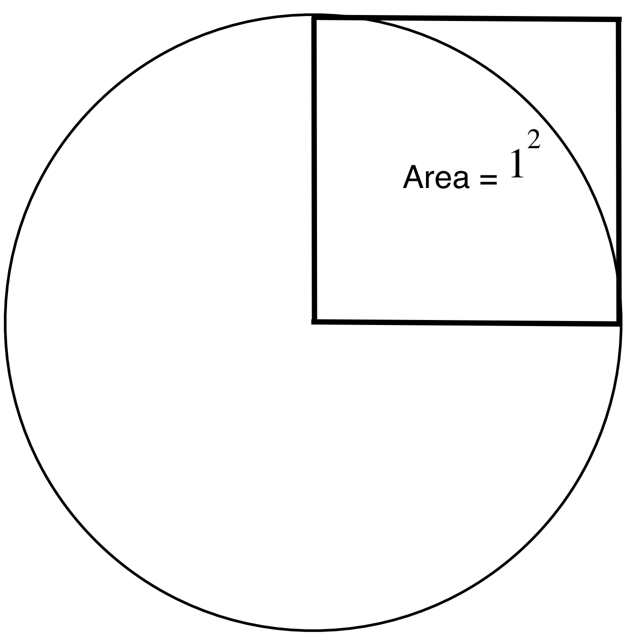
The integral of sinx dx as x goes from 0 to pi/2 is 1.

Based on a unit circle interpretation of this integral, this seems dissonant.

Let's start from the beginning. In a unit circle with a varying angle or radian length (x), the length of the side in the right triangle formed opposite the inscribed angle is the value of sinx. The length of side adjacent to the angle formed by the two radii is the value of cosx. The sine and cosine functions are important because they represent the vertical and horizontal components of the hypotenuse. This means that for a given vector, the sine of the direction times the magnitude of the vector is the component of the vector in the vertical direction. Likewise for the horizontal component and cosine. This is the introduction to the problem. I'm attempting to redefine anything that has to do with my problem for the sake of clarification.



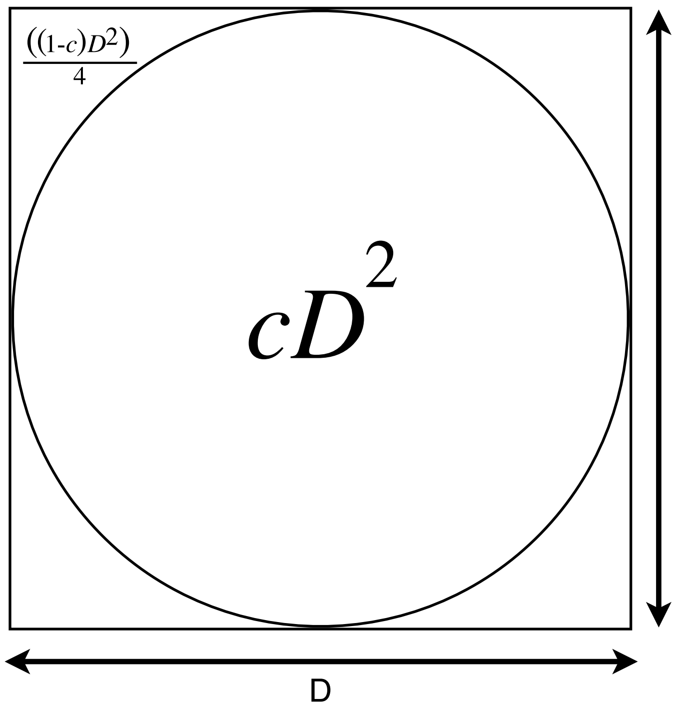
For each incremental change of θ, we have a responding change in the value of sine θ. If, for all incremental changes in θ, we add the corresponding length of sine θ, it <u>seems</u> we should get the area of the sector $\frac{\pi}{2}$. This verbal description is equivalent to $\int_0^{\frac{\pi}{2}} sin\theta \, d\theta$. However, $\int_0^{\frac{\pi}{2}} sin\theta \, d\theta = 1$, therefore it does not represent the area of the sector $\frac{\pi}{2}$. A visual proof of the previous statement can be seen below. I'm currently not sure why the above integral is equal to 1 and not $\frac{\pi}{4}$. Also note the equation for the unit circle in cartesian coordinate system is $x^2+y^2=1$.



The conventional equation for calculating sector areas is: $\frac{\pi r^2}{360} \cdot \theta$. This may be rewritten as: $\pi \cdot r^2 \cdot \frac{\theta}{360}$ So, one multiplies πr^2 by the ratio of the arc size to the circle. Why this value?

Well, for a line segment *a*, and a curve *b* starting and ending at the endpoints of *a,* and each point of which is equidistant to the midpoint of *a*, the ratio between the lengths of *a* and *b* is $\frac{\pi}{2}$. It then stands to reason that the relationship between this line segment and a fully circumscribing curve is π.

The Problem



$$c=\frac{\pi}{4}$$

Therefore, an apt definition of pi is the ratio between the diameter of a circle and its circumference.

$$D^2 \cdot x=\pi \cdot \left(\frac{D}{2}\right)^2$$

Given d, x=

1, .7853. This means that if given a square with sides 1, the area of the circle with diameter 1 will be the area of the square times .7853.
2 (unit circle), .7853

This continues, which means, **for any square, the area of the circle with a diameter equal to the square's side length is the area of the square times** $\frac{\pi}{4}$**.**

Let's now denote $\frac{\pi}{4}$ as *c*.

This leads us back to our original question: why does πr^2 give the area of a circle? Well, knowing *c* is the constant we multiply the area of a square by to find the area of a circle with a diameter equal to the length of the square, we may rewrite the equation.

$$cD^2=4c\left(\frac{D^2}{4}\right)=\pi \cdot \left(\frac{D^2}{4}\right)=\pi \cdot \left(\frac{D}{2}\right)^2=\pi r^2$$

I'm unsure if this is a cyclical definition, and therefore not a valid one.

$\sin^2(0)=0 \; and \; \sin^2\left(\frac{\pi}{2}\right)=1$, thus the approximate average value of $\sin^2$ on the interval $\left[0,\frac{\pi}{2}\right]$ is .5. So, to calculate the area under the curve, we multiply $.5 \cdot \frac{\pi}{2}$ which is the area of the sector $\left[0,\frac{\pi}{2}\right]$ in the unit circle.

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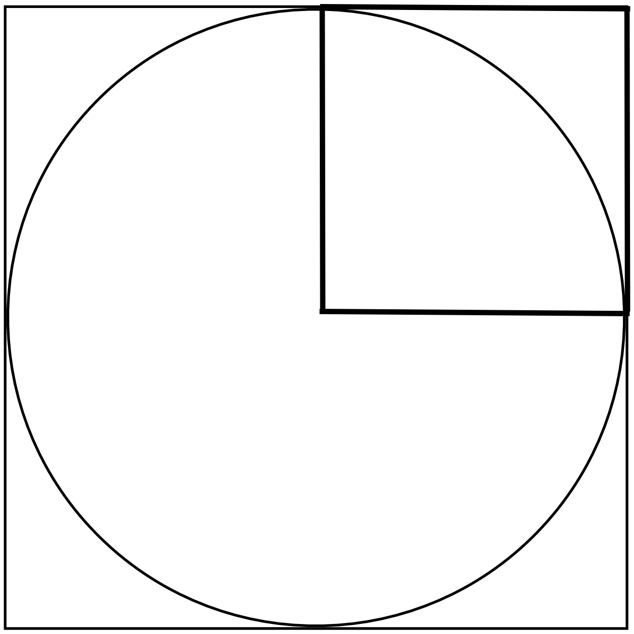
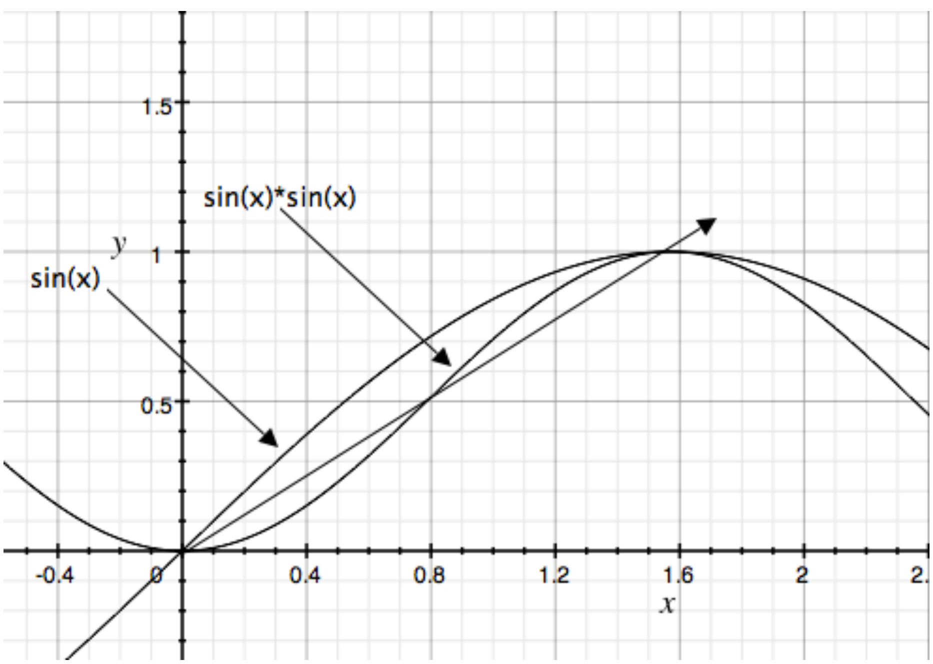


The question is: Why is the integral of sinx on the interval 0..pi/2 equal the area of the darkened square, and not the sector of the circle corresponding to the same radian measure?



$$\sin\left(\operatorname{atan}\left(\frac{y}{x}\right)\right)^2 + \cos\left(\operatorname{atan}\left(\frac{y}{x}\right)\right)^2 = 1$$

This can be done by finding theta's equivalent in terms of y and x.