Notation: m_A and m_B are the masses of the objects A and B, respectively. T is tension, F is force. The x-axis is horizontal and increasing to the right, and the y-axis is vertical and increasing upwards.

You're asked for acceleration, so you know at some point that you'll be dealing with $F = ma$. Assumptions:

1. No friction, anywhere.
2. The rope is massless.
3. The pulleys are massless.
4. Gravity is constant in this region.
5. The rope never goes slack.

First, note that these assumptions imply that the tension in the rope is constant everywhere, so I'll neglect any subscripts on the tension T.

If you do some force diagrams on the two boxes, you'll get two equations:

Box A:

$$\Sigma F = 2T\hat{x} = m_A\vec{a}_A = m_A a_A \hat{x} \tag{1}$$

Box B:

$$\Sigma F = T\hat{y} - m_B g\hat{y} = m_B\vec{a}_B = m_B a_B \hat{y} \tag{2}$$

The unit vectors aren't necessary, they're just there for completeness and so you know explicitly what directions the accelerations will be. From now on, no more directions. Also, I've used g as a positive contant and chosen my force directions (signs) appropriately (a_B will be downwards in an upwards coordinate system, so it will be negative).

Now, you don't care about T, so let's eliminate those:

$$m_A a_A = 2m_B(a_B + g) \tag{3}$$

Just to check some units at this point, you have mass and acceleration on the left and mass and acceleration on the right. Checking dimensions is never a bad idea and can catch mistakes early.

So we need a way to relate the two equations. Our force system had two equations and three unknowns (a_A, a_B, and T), so we need another equation. The missing link comes from thinking about how the rope behaves. For every meter that box A moves to the right, how much does box B have to move? I assert that if A moves 1 unit, then B will move 2 units. This comes from the idea that moving A 1 unit causes 1 unit of rope above the pulley and 1 unit of rope below the pulley to become available, so B will take up the slack. You could also do the opposite way (if B moves 1 unit, how much does A move), but that one is harder for me to think about. This is why I included assumption 5.

So, we have $\Delta x_B = 2\Delta x_A$. Differentiate both sides to accelerations, and you get $a_B = 2a_A$. Cute. But! Remember to account for the signs– we're not paying attention to direction! In truth, A moving 1 unit will cause B to move -2 units, in our coordinate system. Very tricky. So $a_B = -2a_A$.

Plug and chug:

$$m_A a_A = 2m_B(-2a_A + g)$$
$$a_A(m_A + 4m_B) = 2m_B g$$

$$a_A = \frac{2m_B}{m_A + 4m_B} g \tag{4}$$

and

$$a_B = \frac{-4m_B}{m_A + 4m_B} g \tag{5}$$

Let's do some sanity checks. If m_B is zero, then nothing should move, and (4) agrees with that. If m_A is zero, then, well, funny things happen, and equation (1) doesn't make sense anymore. Physically, probably it'd mean that the rope would go slack or something, and that's not okay, so we can ignore this case.

Note that the coefficient can never be greater than 1, so A will never accelerate faster than g. Also a good sign.

For (5), if we let m_A go to zero, then we just recover $a_B = -g$, which makes perfect sense. If we let m_B go small, then we get a small a_B, which also makes sense, as a small mass pulling a big mass will have a small acceleration.

If we had missed the negative sign in $a_B = -2a_A$, then the addition sign in the denominator of (4) would be a subtraction sign, and a_A could go negative, which doesn't make sense. So it's good that we noticed it.

Bonus! Let's do some energy. Say that we start it out with B at height 0 and both masses initially at rest and let it go for some time t. Then $v_a = a_A t$:

$$v_A = \frac{2m_B}{m_A + 4m_B} gt$$

and

$$v_B = \frac{-4m_B}{m_A + 4m_B} gt$$

By the work-energy theorem, the work that gravity does on B will equal the kinetic energy gained by the masses. If we messed up our accelerations, then the work-energy theorem will tell us! The work done is given by

$$W = \int_{y_0}^{y_1} \vec{F} \cdot dl = m_B g h$$

where $h = \Delta y = \frac{1}{2} a_B t^2$. The force is downwards and we're integrating downwards, so the dot product is positive, so positive work is done on the system by gravity. Note that since a_B is in the negative y direction, we must remember that mg is negative as well to keep the work done positive. So we'll put a minus sign on the expression for work to reflect that.

So:

$$W \stackrel{?}{=} \frac{1}{2}(m_A v_A^2 + m_B v_B^2)$$

$$-\frac{1}{2} m_B g a_B t^2 \stackrel{?}{=} \frac{1}{2}(m_A v_A^2 + m_B v_B^2)$$

$$-m_B g a_B t^2 \stackrel{?}{=} m_A (a_A t)^2 + m_B (a_B t)^2$$

$$-m_B g a_B \stackrel{?}{=} m_A a_A^2 + m_B a_B^2$$

(Here, time cancels out– *very* cool. But of course, it *has* to, or else the WE theorem would not (in general) hold for all times.)

$$2 m_B g a_A \stackrel{?}{=} m_A a_A^2 + m_B (-2a_A)^2$$

$$2 m_B g \stackrel{?}{=} m_A a_A + 4 m_B a_A$$

$$2 m_B g \stackrel{?}{=} (m_A + 4 m_B) a_A$$

$$2 m_B g \stackrel{?}{=} (m_A + 4 m_B) \frac{2 m_B}{m_A + 4 m_B} g$$

$$2 m_B g = 2 m_B g \ \checkmark$$

So it's consistent! We did some sanity checks on our accelerations and they seemed reasonable, and we checked it with a completely different bit of physics and it still seems to work out. We can be reasonably confident with it.