Investigating the Discriminant and the Quadratic Formula

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For an equation of the form $ax^2 + bx + c = 0$, the roots of the equation (the values of x such that the statement is valid) is apparently given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To prevent unnecessary computation, we can first determine whether these roots exist by checking to see if the binomial under the radical evaluates to a positive or negative number. That is, check to see if $b^2 - 4ac \geq 0$.

It is said by master Tara that if the discriminate is equal to 0, then the quadratic has one root, one solution.

Let's first examine why it is that $b^2 - 4ac$ determines whether or not the roots exist. The c term quite obviously has an affect on whether the parabola intersects the x-axis. The c term moves the parabola up constantly and predictably. So for any chosen a and b, c alone can make the difference between real and imaginary roots.

The idea for visualizing the last two coefficients is to evaluate each of the terms you're not currently interested in, and then apply the term you are interested in.

For consecutive changes of b, the vertex of the parabola for $b+1$ is the point on b where the y value is the least compared to the x value.