

# Homework 7

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3.5.1 (a)

We can write  $\{a^m b^n : m \neq n\}$  as the union of the following two context-free languages:  $\{a^m b^n : m > n\} \cup \{a^m b^n : m < n\}$ .

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So we take our language  $L$  and intersect it with the language  $a^* c a^* c a^*$ , if  $L$  is context free we know that this intersection should also be context free. However, a string of the form  $a^n c a^n c a^n$  is in the intersection of the two languages and we know, in applying the pumping lemma, that no choice of  $v$  and  $y$  can result in a string that is in the language because choosing  $v$  and  $y$  to be the two  $c$ 's in the language will obviously lead to a string that is not in the language because the string must contain exactly two  $c$ 's, and one may not choose  $v$  and  $y$  to contain any  $a$ 's because doing so will result in an unequal number of  $a$ 's on the remaining side of the  $c$ 's (that is, you cannot pump all three  $a$ 's; only two of them) and so we must conclude that any pumped string would not be in the intersection. Therefore, because the pumped string is not in the intersection this implies that the intersection is not context free which implies that  $L$  is not context free.

3.

We know that  $\{a^m b a^n b a^p : m = n \text{ or } n = p \text{ or } m = p\}$  is context-free because it can be written as the union of the following three context-free languages:  $\{a^m b a^n b a^p : m = n\} \cup \{a^m b a^n b a^p : m = p\} \cup \{a^m b a^n b a^p : n = p\}$